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Bayesian Regression Estimation with Balanced Weighted Precision Loss Function: Cross-Sectional-Qualitative Studies

Dengeli Ağırlıklı Hassas Fonksiyon Kaybı ile Bayes Regresyon Tahmini: Kesitsel ve Nitel Çalışma

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ABSTRACT Objective: The main aim of this study is to derive the alternative risk and loss function for Bayesian paradigm. Bayesian Decision making is an integral part of Bayesian inference which had for long overshadowed due to the adoption of frequentist performance metric, and Bayesian inference is probabilistic in nature, treating it in classical paradigm would lead to poor performances. This study seeks to examine critically and introduce *weight* which eventually makes Bayesian estimates compare favourably well with classical estimates. **Material and Methods:** Balanced weighted precision loss function was adopted and described; it is an extraction of precision of estimates from balanced loss function which ordinarily combined goodness of fit and precision of estimates. The goodness of fit criterion measures the quality of data while the precision of estimates measures the quality of inferences, combining the two criteria may lead to loss of information as each criterion has its specific role in both classical and Bayesian paradigms. **Results:** Weighted quadratic loss to measure the precision of estimates of Posterior mean and Bayes estimate were constructed as a standard metric. The study established the estimation characteristics under weighted quadratic loss function which makes Bayesian inference compare favourably well with other estimators. **Conclusion:** It is therefore recommended that weighted quadratic loss function of assessment criteria of both posterior mean and Bayes estimates is of importance for correct comparison.

Keywords: Prior; precision; loss function; posterior mean and Bayes estimates

ÖZET Amaç: Bu çalışmanın asıl amacı, Bayes paradigması için alternatif risk ve fonksiyon kaybı türetmektir. Bayes karar verme, sıklıkçı performans ölçütlerinin benimsenmesi nedeniyle uzun süre gölgede kalmış olan Bayes çıkarımın ayrılmaz bir parçasıdır ve Bayes çıkarım doğası gereği olasılıklıdır, klasik paradigma ile ele alınması düşük performanslara yol açacaktır. Bu çalışma, Bayes tahminleri klasik tahminlerle olumlu karşılaştıran ağırlığı tanıtmayı ve eleştirel bir bakış açısıyla tanıtmayı amaçlamaktadır. **Gereç ve Yöntemler:** Dengeli ağırlıklı hassas fonksiyon kaybı benimsenmiş ve tanımlanmıştır; uyumun iyiliği ve tahminlerin kesinliğinin kombinasyonu olan dengeli fonksiyon kaybındaki tahminlerin kesinliğinin çıkarımıdır. Tahminlerin kesinliği çıkarımların kalitesini ölçerken uyumun iyiliği kriterleri verilerin kalitesini ölçer. Her kriterin kendine özgü spesifik rolü olduğundan iki kriterin birleştirilmesi bilgi kaybına yol açabilir. **Bulgular:** Posterior ortalamanın tahminlerinin kesinliğini ölçmek için ağırlıklı kuadratik kayıp ve Bayes tahmini standart bir ölçüm olarak oluşturulmuştur. Çalışma, diğer tahminlerle Bayes çıkarımlarını olumlu bir şekilde karşılaştıran ağırlıklı kuadratik fonksiyon kaybı altında tahmin karakteristiklerini ortaya koymuştur. **Sonuç:** Bu sebeple, posterior ortalama ve Bayes tahminlerinin ağırlıklı kuadratik fonksiyon kaybı değerlendirme kriterlerinin doğru karşılaştırma için önemi vurgulanmıştır.

Anahtar kelimeler: Öncelik; kesinlik; fonksiyon kaybı; posterior ortalama ve Bayes tahminleri

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It was observed in the literature that not at all times prior distribution is dominated by the likelihood.[1-3] Thus, in Bayesian decision theoretic, a decision has to be made for variety of choices of action “a” subject to the state of nature θ which is unknown.[4] The decision maker’s knowledge is the probability distribution which combines prior knowledge of θ with the information provided by the experiment (likelihood). There is need to choose action $\hat{\theta}$ which minimizes loss over the posterior distribution. There are different results from different choices of prior. The work with reference to point estimate and as well as mean squares error were evaluated, they found out that the authors were disturbed by poor performance of the estimators due to large mean square error (MSE), and that point estimate had large MSE which make it inefficiency.[4-6]

The unbiased estimator should not be adopted as criterion due to large MSE, thus, the use of posterior mean might yield inefficient estimator and that Bayesian estimator fails due to infinite Bayes risk, which had large bias due to the posterior distribution with quasi prior.[5] The reason being that the authors failed to weigh the risk function thereby have large bias and MSE, and are also in the category of not adding weight to the bias and risk, both would escalate the criteria and perform poorly as compared with ordinary least squares (OLS).[7,8] In an attempt to improve on this and managed to proposed another balanced loss function called weighted balanced loss function, which attempted to divide the balance loss function (BLF) by the square of the OLS parameters.[9] The duo of goodness of fit that is mean sum of squares of residuals of regression estimation and precision of the estimates which can be obtained from the weighted or unweighted loss function can be simultaneously used as performance metric to evaluate the performance of regression estimator.[7] In line with this assertion, balanced loss function was postulated which makes use of quadratic loss function, that is the combination of sum of squares of residuals and weighted sum of estimation errors. The measures of goodness of fit and measures of precision of estimates to formulate the loss function through which optimal estimates and posterior expected loss were obtained.[9] This study argued that since each of the performance metrics either goodness of fit or precision of estimate has its respective function in evaluating the performance of regression, thus combining them may lead to loss of information or inability to measure what each of them was designed to measure. In line with this, balance precision loss function was proposed which considered only precision of estimate to derive posterior and Bayes expected loss function.

All these efforts may not give accurate or optimal decision and may not be scientifically explained, the main point is just to weighing it. Due to this aged-long problem, this study is trying to fill the gap. The section of the study is arranged as follow: section 1 contains introduction while section 2 examines Expected loss and risks which covered Frequentist, posterior and Bayesian. Balanced Weighted Precision Loss function (BWPLF) is described in section 3, section 4 considered data analysis and interpretation while section 5 contains conclusion.

BALANCED LOSS FUNCTION (BLF)

From literature various authors have adopted weighted balanced loss function and unweighted balanced loss function, but none of them considered BWPLF that specifically meant for precision of estimates. Balanced and unbalanced loss functions were compared, they found out that when goodness of fit is used singlehandedly the $\hat{\beta}$ turned out to be the best choice based on their choosing value of h but when both criteria (goodness of fit and precision of estimates) are used simultaneously, it was observed that $\hat{\beta}$ is optimal if the h is greater than specified f but if the h is less than specified f then the optimal falls.[10]

Balanced loss function in the realm of regret loss was examined and established the relationship between quadratic balanced loss and usual quadratic loss, they examined the implication of their study with respect to Stein-rule estimators, they examined the regret loss with reference to several non-quadratic balanced loss function.[11] Balanced loss function as criterion to evaluate the performance of estimates of Posterior mean was adopted and obtained its Bayes estimator using weighted balanced loss function.[12] They concluded that the obtained Poisson mean is admissible. Weighted squared error loss, entropy loss and weighted entropy loss to examine the characteristics of estimators were adopted by adopted.[13-15]

Balanced loss function was adopted in the study and concluded that both the classical and Bayesian depicted similar results which is only a reflection of precision of estimates.[16] Their study led to generalized improved Bayes estimator under balanced loss function. Constrained Bayes and empirical Bayes estimators under balanced loss functions was derived.[17] Thus, their study established that both estimators have their individual usefulness particularly if comparison is taken place between them. The target estimator has certain optimal characteristics under balanced loss function and they established relationship between optimal actions derived under balanced and unbalanced losses function.[18,19] Considered the problem of estimating a continuous distribution function F and $\tau(F)$ under a variety of loss functions.[20] Examined estimations of a normal mean under balanced loss functions, examined Bayesian estimation and measured it with different loss functions and were able to develop new estimator under different loss functions.[21,22] Opined that Bayesian estimator under loss function outperformed other classical estimators, this study therefore postulated both Bayes estimator and posterior mean under the unbalanced loss function called sliced loss function by single-handedly considering precision of estimate.

EXPECTED LOSS AND RISKS

Theorem 1: Let $D = (X_1, X_2, \dots, X_n)$ be set of data with probability distribution $P(\beta)$ with $\beta \sim \pi$, π is the prior and $\hat{\beta} = \delta(D)$, thus, $\delta(D)$ can be expressed as $\delta(D)$, depending on the nature of data. The loss function can be expressed as $L(\beta, \hat{\beta}) = L(\beta, \delta(D))$, the two random variables are minimized by taking its expected value. There are two possible paths: average of data (frequentist) and average of theta (prior) Bayesian.

FREQUENTIST EXPECTED LOSS

Def 1: the loss function of a decision rule $\delta(D)$ can be expressed as:

$$E\big(L\big(\beta, \delta(D)\big)|\beta\big)$$

This is average on data D conditional on parameter β, the belief of frequentist is that there is adequate knowledge of data, and it does not make sense to average on prior distribution that is unknown, and there is no justification for using prior distribution for making decision or take action.

Def 2: the risk function of a decision rule $\delta(D)$ is obtained as:

$$R(\beta, \delta) = E\big(L\big(\beta, \delta(D)\big)|\beta\big) = \int_{\mathcal{A}} L\big(\beta, \hat{\beta}\big)\pi(\beta|D)d\beta \qquad (1)$$

The frequentist risk function is the integral of loss function and likelihood

POSTERIOR EXPECTED LOSS AND RISK

$$\rho\big(\pi(\beta|D), \hat{\beta}\big) = \mathrm{E}\big(L\big(\beta, \hat{\beta}\big)|D\big) = \int_{\Theta} L\big(\beta, \hat{\beta}\big)\pi(\beta|D)d\beta \qquad (2)$$

Adopting quadratic loss function, we have

Let β denote the set of nature S and $\hat{\beta}$ represent the action a having real value loss function of $\big(\beta, \hat{\beta}\big)$, $D = X_1, X_2, \dots X_N$ set of observations, β is a random variable, the distribution of D depends on β, the optimal decision is to chose $\hat{\beta}$ that will minimize the posterior expected loss.

$$\int_{\Theta} L\big(\beta, \hat{\beta}\big)\pi(\beta|D)d\beta = \int_{\Theta} \big(\beta - \hat{\beta}\big)^2 \pi(\beta|D)d\beta \qquad (3)$$

Solve for $\hat{\beta}$ we have

$$\hat{\beta} = E(\beta)\pi(\beta|D)$$

BAYESIAN EXPECTED LOSS FUNCTION

Def 3: If $\boldsymbol{\pi}^*(\boldsymbol{\beta})$ is the prior probability density of $\boldsymbol{\beta}$ during the time of decision making, then the Bayesian expected loss of an action $\widehat{\boldsymbol{\beta}}$ is $\boldsymbol{\rho}(\boldsymbol{\pi}^*, \widehat{\boldsymbol{\beta}}) = \boldsymbol{E}^{\boldsymbol{\pi}^*}\boldsymbol{L}(\boldsymbol{\beta}, \widehat{\boldsymbol{\beta}}) = \int_{\boldsymbol{\Theta}} \boldsymbol{L}(\boldsymbol{\beta}, \widehat{\boldsymbol{\beta}})\boldsymbol{\pi}^*(\boldsymbol{\beta})\boldsymbol{d}(\boldsymbol{\beta})$

Let $\boldsymbol{\delta}(\boldsymbol{D})$ be the decision rule associated with prior density $\boldsymbol{\pi}(\boldsymbol{\beta})$, the expected value of the loss function () from the joint probability density function $(\boldsymbol{X}, \boldsymbol{\beta})$ is referred to as Bayes risk $\boldsymbol{r}_{\boldsymbol{\pi}}(\widehat{\boldsymbol{\beta}}|\boldsymbol{D})$ which is expressed as : $\boldsymbol{r}_{\boldsymbol{\pi}}(\widehat{\boldsymbol{\beta}}|\boldsymbol{D}) = \boldsymbol{E}_{\boldsymbol{\pi}}\left[\boldsymbol{E}\left[\boldsymbol{L}(\boldsymbol{\beta}, \widehat{\boldsymbol{\beta}})|\boldsymbol{\beta}\right]\right] = \boldsymbol{E}_{\boldsymbol{\pi}}\left[\boldsymbol{R}(\boldsymbol{\beta}, \widehat{\boldsymbol{\beta}})\right]$

Thus, the Bayes rule is the procedures that minimize the Bayes risk. The posterior risk of an action $\boldsymbol{a}$ is the expected loss from taking action $\boldsymbol{a}$ under the posterior distribution as $\boldsymbol{\pi}(\boldsymbol{\beta}|\boldsymbol{D})$

$$\boldsymbol{r}(\widehat{\boldsymbol{\beta}}|\boldsymbol{D}) = \boldsymbol{E}_{\boldsymbol{\pi}(\boldsymbol{\beta}|\boldsymbol{D})}\left[\boldsymbol{L}(\boldsymbol{\beta}, \widehat{\boldsymbol{\beta}})\right]$$

$\boldsymbol{L}: \boldsymbol{\Theta} \times \boldsymbol{\mathcal{A}} \rightarrow \mathbb{R}$ is the loss function of loss incurred as a result of the action $\boldsymbol{a}$ employed with state of the nature $\boldsymbol{\beta}$. The risk function is expressed by

$$\boldsymbol{R}(\boldsymbol{\theta}, \boldsymbol{D}) = \int_{\mathbb{R}\widehat{\boldsymbol{\beta}}} \boldsymbol{L}(\boldsymbol{\beta}, \boldsymbol{D})\boldsymbol{f}(\boldsymbol{D}|\boldsymbol{\beta})\boldsymbol{dD} \quad (4)$$

Bayes risk can be expressed by

$$\boldsymbol{r}(\boldsymbol{\pi}, \boldsymbol{D}) = \int_{\mathbb{R}\widehat{\boldsymbol{\beta}}} \boldsymbol{R}(\boldsymbol{\beta}, \boldsymbol{D})\boldsymbol{\pi}(\boldsymbol{\beta})\boldsymbol{d\beta} \quad (5)$$

$$r(\pi, D) = \int_{\Theta}\int_{\mathbb{R}\widehat{\beta}} L(\beta, D)f(D|\beta)dD\pi(\beta)d\beta$$

$$r(\pi, D) = \int_{\mathbb{R}\widehat{\beta}}\int_{\Theta} L(\beta, D)\ \pi(\beta|D)d\beta$$

$$r(\pi, D) = \int_{\mathbb{R}\widehat{\beta}}\int_{\Theta} L(\beta, D)\ \pi(\beta|D)d\beta$$

which is equivalent to minimizing posterior expected loss. Note that Θ is the all-possible state of nature that is unknown quantities, it can be regarded as parameter space of which θ is the parameter that is $\theta\epsilon\Theta$. Thus, a is the action or decision of which $\mathcal{A}$ is the set of all possible actions, hence $a\ \epsilon\mathcal{A}$.

BALANCED WEIGHTED PRECISION LOSS FUNCTION (BWPLF)

Let $y = X\beta + u$ be a linear regression model with y as a vector of observations of the dependent variable, X is an $n \times k$ matrix of observations with rank $k < n$, β is a vector of unknown parameters and u is a vector of disturbance error term in line with, the posterior pdf for β has the property of finite second moment. BLF has been used over decades in the literature as a performance metric in a Bayesian paradigm.[7] Thus, it combined goodness of fit and precision of estimate as in, it may be impossible to make comparison between Bayesian and frequentist if both are not measured in the same frequency.[7]

Let $u'u = (y - X\beta)'(y - X\beta)$ (6)

Simplify rhs of $u'u$ further we have

$$= (y'y - 2\beta'X'y + \beta'X'X\beta)$$

$$= y'y - 2\beta'X'y + \beta'X'X\beta - 2((X'X)^{-1}X'y)'X'y + ((X'X)^{-1}X'y)'X'X((X'X)^{-1}X'y)$$

$$= (y - Xb)'(y - Xb) + b'X'Xb + \ \beta'X'X\beta - 2\beta'X'Xb$$

$$= \left(\hat{\sigma}(n - k) + (\beta - b)'X'X(\beta - b)\right)$$

$$b = (X'X)^{-1}X'y$$

Expressing eq (6) as BLF in order to estimate posterior pdf of $\dot{\beta}$, we have:

$$L_B(\hat{\beta}, \beta) = w(y - X\hat{\beta})'(y - X\hat{\beta}) + (1 - w)(\bar{\beta} - \beta)'(\bar{\beta} - \beta) \quad (7)$$

The equation has two parts criteria, the first path represents goodness of fit while the other part denotes precision of estimate. Suffice to say the study will consider the second path, with $0 \leq w \leq 1$ and $\bar{\beta}$ is the estimate of β, we will obtain $\bar{\beta}$ which minimizes posterior expected loss, and $\breve{\beta}$ the Bayesian estimate that minimizes posterior expected loss.

Thus, w and $1 - w$ are defined as:

$$1 - w = \frac{\hat{\tau}^2}{\hat{\tau}^2 + (\hat{\sigma}^2/n)}$$

$$w = \frac{\hat{\sigma}^2/n}{\hat{\tau}^2 + (\hat{\sigma}^2/n)}$$

$$\breve{\beta} = w\hat{\beta} + (1 - w)\bar{\beta} \tag{8a}$$

It should be noted that Bayes estimate $\breve{\beta}$ is the weighted average of OLS $\hat{\beta}$ and posterior mean.[5]

$$\hat{\beta} = (X'X)^{-1}X'y$$

$\bar{\beta}$ is the posterior mean of β

$$\bar{\beta} = a\hat{\beta} + (1 - a)\mathbb{B} \tag{8b}$$

In eq.(8b) above $\mathbb{B}$ is the prior parameters, thus in Posterior expected loss, there is no justification to include measure of goodness of fit in our BWPLF, since the Bayes risk function is to be compared with frequentist risk as well as other frequentist criteria

with the omission of goodness of fit we have a criterion

$$\pi(\beta) = (\beta - \mathbb{B})'\Sigma^{-1}(\beta - \mathbb{B}) \tag{9}$$

$$E_{\mathbb{B}}L_B(\hat{\beta}, \beta) = \left[(1 - w)(\bar{\beta} - \beta)'(\bar{\beta}'1)^{-1}(\bar{\beta} - \beta)\right] \tag{10}$$

$$E_{\mathbb{B}}L_B(\breve{\beta}, \beta) = \left[(1 - w)(\breve{\beta} - \beta)'(\bar{\beta}'1)^{-1}(\breve{\beta} - \beta)\right] \tag{11}$$

Note that there may be need to remove w form the equation above except if it will be inclusive in the frequentist criteria. Besides, we weighted the equation (10) and (11) above since prior is inherent in both posterior mean and Bayes estimate.

TABLE 1: Average operating characteristics.

Average operating characteristic	Frequentist	Posterior mean	Bayes estimate
Mean	$\hat{\beta} = (X'X)^{-1}X'y$	$\bar{\beta} = \alpha\tilde{\beta} + (1-\alpha)\mathbb{B}$	$\breve{\beta} = \alpha\bar{\beta} + (1-\alpha)\mathbb{B}$
Bias	$\hat{\beta} - \beta$	$\bar{\beta} - \beta$	$(\breve{\beta} - \beta)$
Loss function	$(\hat{\beta} - \beta)'(\hat{\beta} - \beta)$	$(1-\alpha)(\bar{\beta} - \beta)'(\bar{\beta} - \beta)$	$(1-w)(\breve{\beta} - \beta)'(\breve{\beta} - \beta)$
Bayes risk	$(\hat{\beta} - \beta)'(\hat{\beta} - \beta)$	$\frac{(1-\alpha)(\bar{\beta} - \beta)'(\bar{\beta} - \beta)}{(\bar{\beta}'1)^{-1}}$	$\frac{(1-w)(\breve{\beta} - \beta)'(\breve{\beta} - \beta)}{(\breve{\beta}'1)^{-1}}$

The Table 1 presented average operating characteristics of risk and loss function. The above average operating characteristics omitted a or w, there is no justification of imposing it and that it can be added, but care must be taken to add it thoroughly including OLS. This weight of the loss function makes Bayes risk outperformed posterior risk and frequentist risk. This will correct age long assertion that made having large MSE in their study.[5]

DATA ANALYSIS, RESULTS AND INTERPRETATION

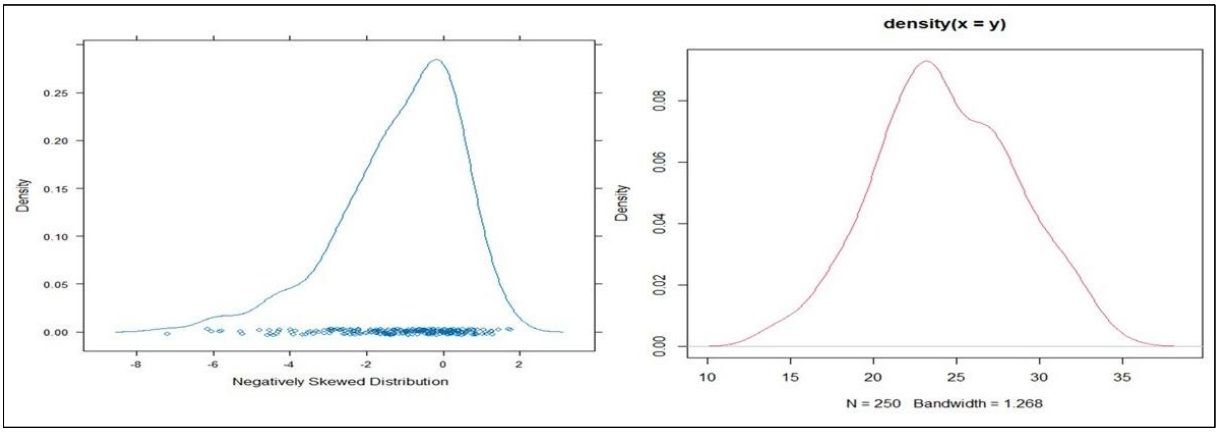

FIGURE 1: Depicts the density of negatively skew-normal error and dependent variable y.

From Figure 1 above, the data generated with negatively skew-normal error with sample size 250, this is common in health research. The density of the error has impact on the dependent variable, the data was analysed with both classical and Bayesian frameworks to ascertain the effectiveness of loss and risk function proposed in the study.

TABLE 2: Depicts negatively skew-normal error structure dataset in classical and Bayesian frameworks.

	Classical		Bayesian OLS		Bayesian stein	
Negatively skewed data	**OLS**	**Stein-rule**	**Posterior mean**	**Bayes estimate**	**Posterior mean**	**Bayes estimate**
Mean	0.9348	0.9192	1.2488	1.2485	1.2502	1.2504
	1.9993	2.0012	1.9917	1.9918	1.9915	1.9915
	0.8006	0.7998	0.6924	0.6924	0.6924	0.6924
	0.2995	0.3001	0.2829	0.2829	0.2828	0.2828
	2.0993	2.1005	2.0781	2.0781	2.0780	2.0780
	1.0999	1.0997	1.0643	1.0643	1.0643	1.0643
Bias	-0.2652	-0.2808	0.0332	0.0329	0.0341	0.0343
	-0.0007	0.0012	-0.0056	-0.0056	-0.0058	-0.0058
	0.0006	-0.0002	-0.0731	-0.0731	-0.0731	-0.0731
	-0.0005	0.0001	-0.0116	-0.0116	-0.0117	-0.0117
	-0.0007	0.0005	-0.0149	-0.0149	-0.0149	-0.0149
	-0.0001	-0.0002	-0.0243	-0.0243	-0.0243	-0.0243
Loss function	0.07033	0.07886	0.010926	0.010908	0.011028	0.011042
Bayes risk	0.00972	0.01092	0.001472	0.001469	0.001499	0.001500

OLS: Ordinary least squares.

In Table 2 above, the error structure is negatively skew-normal as it depicted in Figure 1 above, the outcome of the study pointed out that Bayesian framework outperformed classical paradigm both in loss function and Bayes risk, this affirms the efficiency of our proposed BWPLF.

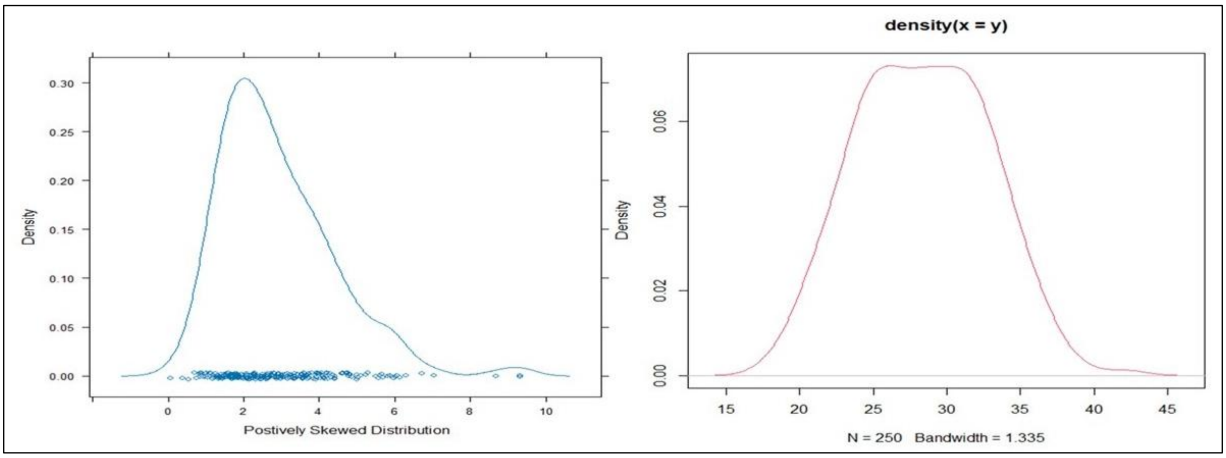

FIGURE 2: Depicts the density of positively skew-normal error and dependent variable y.

From Figure 2 above, the data generated with positively skew-normal error with sample size 250, this is common in health research. The density of the error has impact on the dependent variable, the data was analysed with both classical and Bayesian frameworks to ascertain the effectiveness of loss and risk function proposed in the study.

TABLE 3: Depicts positively skew-normal error structure dataset in classical and Bayesian frameworks.

	Classical		Bayesian OLS		Bayesian stein	
Positively skewed data	**OLS**	**Stein**	**Posterior mean**	**Bayes estimate**	**Posterior mean**	**Bayes estimate**
Mean	4.0463	4.0575	3.0529	3.0524	3.0552	3.0556
	2.0007	1.9998	2.1645	2.1646	2.1642	2.1641
	0.7998	0.8003	0.8501	0.8500	0.8501	0.8500
	0.3007	0.2996	0.2882	0.2883	0.2882	0.2882
	2.1009	2.0999	2.0692	2.0692	2.0691	2.0690
	1.1000	1.1001	1.1484	1.1484	1.1484	1.1484
Bias	2.8463	2.8575	1.2449	1.2446	1.2465	1.2467
	0.0007	-0.0002	0.1105	0.1106	0.1103	0.1103
	-0.0001	0.0003	0.0336	0.0336	0.0336	0.0336
	0.0007	-0.0004	-0.0079	-0.0079	-0.0079	-0.0079
	0.0009	-6.086-05	-0.0207	-0.0207	-0.0208	-0.0208
	4.98e-05	7.58e-05	0.0325	0.0325	0.0325	0.0325
Loss function	8.1016	8.1654	2.3288	2.3277	2.3346	2.3354
Bayes risk	0.78287	0.78838	0.2410	0.2409	0.2438	0.2439

OLS: Ordinary least squares.

In Table 3 above, the error structure is positively skew-normal as it depicted in Figure 2 above, the outcome of the study pointed out that Bayesian framework outperformed classical paradigm both in loss function and Bayes risk, this affirms the efficiency of our proposed BWPLF.

CONCLUSION

BWPLF that captured precision of estimation was examined to develop performance metrics for posterior and Bayesian estimates. Conventional BLF was derived from sum of squares of error of which BWPLF was extracted. However, aged-long inefficient metrics had been weighted, thereby making it efficient and comparatively well with other estimators. The performance metrics were established with regression parameters for easy understanding. The study affirmed the efficiency of our proposed BWPLF in comparison with classical framework. Both OLS and Stein -rule estimators were compared with Bayesian paradigms, the study observed better performances of Bayesian paradigm in comparison with classical frameworks.

Source of Finance

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Conflict of Interest

No conflicts of interest between the authors and/or family members of the scientific and medical committee members or members of the potential conflicts of interest, counseling, expertise, working conditions, share holding and similar situations in any firm.

Authorship Contributions

Idea/Concept: *Isiaka Oloyede;* ***Design:*** *Isiaka Oloyede, Ipinyomi Rueben;* ***Control/Supervision:*** *Ipinyomi Rueben;* ***Literature Review:*** *Isiaka Oloyede, Ipinyomi Rueben;* ***Writing the Article:*** *Isiaka Oloyede;* ***Critical Review:*** *Isiaka Oloyede, Ipinyomi Rueben;* ***References and Fundings:*** *Isiaka Oloyede, Ipinyomi Rueben.*

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