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A New Modification of Ridge Parameter for Regression Problems: A Monte Carlo Simulation Study

Regresyon Problemleri için Ridge Parametresinin Yeni Bir Uyarlaması: Monte Carlo Simülasyon Çalışması

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ABSTRACT Objective: In multiple linear regression, there should be no correlation between explanatory variables. When there is a high correlation between two or more explanatory variables, it is called a multicollinearity problem. In the case of multicollinearity, the effect of independent variables on the dependent variable is parsed. Ridge regression is one of the commonly used methods in the literature to overcome this problem. The most important issue in ridge regression analysis is the prediction of the ridge parameter. In this study, commonly used estimators in the literature have been investigated and a new estimator has been proposed to estimate the ridge parameter. **Material and methods:** The performance of the proposed estimator was compared in a simulation study. The mean squares error (MSE) criterion was used to compare the performances of the estimators. **Results:** Under various conditions, the proposed estimator was found to perform better than the ordinary least squares estimator (OLS) and other predictors in the literature. For addition, a real data example has been performed by using prostate cancer data. For this data set, the MSE value of the proposed estimator was calculated as smaller than all the other estimators. **Conclusion:** It was found that using ridge regression which is widely used in case of multicollinearity, the proposed estimator for ridge parameter would increase the effectiveness of the potential studies.

Keywords: Linear model; multicollinearity; ordinary least squares estimator; ridge regression; simulation

ÖZET Amaç: Çoklu doğrusal regresyonda açıklayıcı değişkenler arasında korelasyon olmaması gerekir. Eğer iki veya daha fazla açıklayıcı değişken arasında yüksek korelasyon varsa, bu durum çoklu iç ilişki problemi olarak adlandırılır. Çoklu iç ilişki durumunda, bağımsız değişkenlerin bağımlı değişken üzerindeki etkisi ayrıştırılamamaktadır. Ridge regresyon, literatürde bu problemin üstesinden gelmek için yaygın olarak kullanılan yöntemlerden biridir. Ridge regresyon analizinde en önemli problem ridge parametresinin tahminidir. Bu çalışmada, ridge parametresini tahmin etmek için literatürde var olan ve yaygın olarak kullanılan tahmin edicileri araştırılmış ve yeni bir tahmin edici önerilmiştir. **Gereç ve Yöntemler:** Önerilen tahmin edicinin performansı bir simülasyon çalışması yapılarak karşılaştırılmıştır. Tahmin edicilerin performanslarını karşılaştırmak için Hata Kareler Ortalaması (MSE) kriteri kullanılmıştır. **Bulgular:** Çeşitli koşullar altında, önerilen tahmin edicinin, en küçük kareler tahmin edicisinden (OLS) ve literatürdeki diğer tahmin edicilerden daha iyi performans gösterdiği saptanmıştır. Ayrıca Prostat kanseri verisi kullanılarak gerçek veri uygulaması yapılmıştır. Bu veri seti için önerilen tahmin edicinin MSE değeri, diğer tüm tahmin edicilerden küçük olarak hesaplanmıştır. **Sonuç:** Çoklu iç ilişki durumunda yaygın kullanıma sahip ridge regresyonda, ridge parametresi için önerilen tahmin edicinin kullanılmasının yapılacak çalışmaların etkinliğini arttıracağı saptanmıştır.

Anahtar Kelimeler: Doğrusal model; çoklu iç ilişki; sıradan en küçük kareler tahmin edicisi; ridge regresyon; simülasyon

The general expression for the multiple regression model is given below.

$$Y=X\beta+\varepsilon, \tag{1}$$

Here, Y is an $n\times1$ vector of response, $\boldsymbol{\beta}$ is a $p\times1$ vector that includes unknown parameters, X is an $n\times$p matrix of the observed regressors, and ε is an $n\times1$ vector of random errors. The errors are distributed as $N_n(0,\sigma^2 I_n)$, where I_n is the identity matrix of order n. The ordinary least square estimator (OLS) for the regression coefficients β is obtained as given in the following equation,

$$\hat{\beta}_{OLS}=(X'X)^{-1}X'y, \tag{2}$$

and the covariance matrix of $\hat{\beta}_{OLS}$ is obtained as

$$Cov\,(\hat{\beta}_{OLS})=\sigma^2(X'X)^{-1}. \tag{3}$$

$\hat{\beta}_{OLS}$ estimator has well properties. The assumption in multiple linear regression model includes that the explanatory variables are independent. On the other hand, there may be near to strong or strong relationships among the explanatory variables in practice. Such cases cause to the multicollinearity problem in which the independence assumptions are no longer valid. It is not possible to estimate the unique effects of individual variables in the regression equation under the presence of multicollinearity. In addition, the regression coefficients have larger sampling variances which affects both the prediction and inference. Therefore, multicollinearity becomes one of the serious problem in the linear regression analysis. In literature, there are various methods which are used for re-determination of the model and application of alternative regression models to solve this problem. Among them, the "ridge regression" proposed by Hoerl and Kennard is the most popular and useful regression method when faced with multicollinearity problems in real life.[1]

In the problems that include multicollinearity, the variance value of the resulting estimator is determined to be larger and the regression coefficients are different from the actual values. In other words, it is difficult to make valid statistical inferences with these errors. As a solution to this situation, Hoerl and Kennard proposed the ridge estimator by adding a constant k to the diagonal elements of the OLS estimator matrix. The ridge regression estimates some of the relevant parameters by adding some bias but ridge estimators have a smaller variance in this case. The ridge estimator for multiple linear regression is expressed as follows.[1]

$$\hat{\beta}_R=[X'X+k\mathrm{I}]^{-1}X'Y \tag{4}$$

In Equation (4), k denotes the bias parameter and the selection of this parameter affects the performance of the ridge estimator and $k>0$.

The MSE of the ridge estimator $\hat{\beta}_R$ is given as follows

$$MSE\,(\hat{\beta}_R)=\sigma^2\sum_{j=1}^{p}\frac{\lambda_j}{(\lambda_j+k)^2}+k^2\beta'(X'X+k\mathrm{I}p)^{-2}\beta \tag{5}$$

where λ_j's are the descending eigenvalues of $X'X$.

The first term in Equation (5) is the asymptotic variance function which is monotonically decreasing according to the value of k. The second term in the equation is the squared bias function which is monotonically increasing according to the value of k. For a positive value of k,

$$MSE(\hat{\beta}_R) < MSE(\hat{\beta}_{OLS}) \quad (6)$$

$MSE(\hat{\beta}_R)$ depends on σ^2, β and k which are unknown in practice. Thus, the minimization of the MSE values of the ridge estimator depends on the selection of the parameter k which is estimated from real data. Therefore, proper choice of k is an important criterion. There are many studies proposing different methods to estimate the value of k in the literature. First, Hoerl and Kennard proposed a k estimator in 1970.[1] After Hoerl and Kennard, many researchers studied on the ridge parameter estimation problem and proposed different ridge parameter estimators. Most previous studies suggest the ridge parameter estimation problem.[2-8]

The purpose of this study is to propose an estimation method for the ridge parameter k as an addition to the current estimation methods in the literature and make a comparison between them according to the MSE criterion.

The article is organized as follows. In Section 2, we present various estimators in the literature and present the proposed estimator. In Section 3, the performance of the new estimator for choosing ridge parameter is evaluated by the simulation technique in terms of MSE. In Section 4, we analyzed a real data sample to illustrate the benefits of our proposed estimator. Finally, results and discussions are given in Section 5.

MATERIAL AND METHODS

The canonical form of the regression model given in Equation 1 can be expressed in a different formulation. Suppose that there exist an orthogonal matrix D such that $D'CD = \Lambda$, where $C=X'X$, and $\Lambda=$ diag $\lambda_1, \lambda_2, \ldots \lambda_p)$ contains the eigenvalues of the matrix C. The canonical (orthogonal) form of the multiple regression model in Equation (1) is

$$y = X^*\alpha + e, \quad (7)$$

where $X^* = XD$ and $\alpha = D'\beta$. The OLS estimator is expressed as follows

$$\hat{\alpha}_{OLS} = \Lambda^{-1} X^{*\prime} y. \quad (8)$$

When the matrix $X'X$ is ill-conditioned (in the multicollinearity problem), ridge regression representation replaces $X'X$ with $X'X + kI$, $k>0$ and the ridge estimator is written as:

$$\hat{\alpha}_R (X^{*\prime} X^* + K)^{-1} X^{*\prime} y \quad (9)$$

where $K = diag(k_1, k_2, \ldots, k_p)$, $k_i > 0$.

According to Hoerl and Kennard, the value of k_i which minimizes the

$$MSE(\hat{\alpha}_R) = \sigma^2 \sum_{i=1}^{p} \frac{\lambda_j}{(\lambda_j + k_i)^2} + \sum_{i=1}^{p} \frac{k_i^2 \lambda_j}{(\lambda_j + k_i)^2} \quad (10)$$

is

$$HK1 = k_i = \frac{\sigma^2}{\alpha_i^2} \quad (11)$$

where σ^2 stands for the error variance of the multiple regression model, and α_i is the i^{th} element of α.[1] Equation (10) is supposed to give a value of k_i that fully depends on the unknown σ^2 and α_i and it must be estimated from the observed data. They suggested to use the common unbiased estimator $\sigma^2=(y'y-\hat{\alpha}'X'y)/(n-p)$.

Hoerl and Kennard proposed the replacement of σ^2 and α_i by their corresponding unbiased estimators.[1] Since both parameters k_i given in Equation (11) should also be estimated, they suggested the following estimator.

$$HK2=k=\frac{\hat{\sigma}^2}{\alpha^2_{max}} \quad (12)$$

where, α^2_{max} is the maximum element of α. Another estimator proposed by Hoerl and Kennard is given in Equation (13).

$$HK3=\frac{\hat{\sigma}^2}{\hat{\alpha}'\hat{\alpha}}=\frac{\hat{\sigma^2}}{\sum_{i=1}^{p}\alpha_i^2} \quad (13)$$

Theobald proposed a different estimator in 1974.[9]

$$\text{Theobald}=\frac{2\hat{\sigma}^2}{\hat{\alpha}'\hat{\alpha}} \quad (14)$$

HKB estimator suggested by taking the harmonic mean of k_i in Equation in (11).[2]

$$HKB=\frac{p\hat{\sigma}^2}{\hat{\alpha}'\hat{\alpha}}=\frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\alpha_i^2} \quad (15)$$

Later, Lawless and Wang suggested the different estimator by the Bayesian point of view.[10]

$$LW=\frac{p\hat{\sigma}^2}{\hat{\alpha}'X'X\hat{\alpha}}=\frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\lambda_i\alpha_i^2} \quad (16)$$

Hocking, Speed and Lynn proposed the following estimator for k.[11]

$$HSL=\hat{\sigma}^2\frac{\sum_{i=1}^{p}\left(\lambda_i\alpha_i\right)^2}{\left(\sum_{i=1}^{p}\lambda_i\alpha_i^2\right)^2} \quad (17)$$

Kibria proposed 3 different k estimators based on $m_{Kibria}=m_i=\frac{\sigma^2}{\alpha_i^2}$.[12]

$$Kibria1=mean\ (m_i) \quad (18)$$

$$Kibria2=median\ (m_i) \quad (19)$$

$$Kibria3 = \frac{\hat{\sigma}^2}{\left(\prod_{i=1}^{p} \hat{\alpha}_i^2\right)^{1/p}} \tag{20}$$

The *Kibria*2 estimator is proposed for $p \geq 3$.

In 2005, Khalaf and Shukur proposed a new k estimator as a modification of the HK2 estimator given in Equation (12).[13]

$$KS = k = \frac{\lambda_{max}\hat{\sigma}^2}{(n - p - 1)\,\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}_{max}^2} \tag{21}$$

where λ_{max} is the maximum eigenvalue of the matrix $X'X$.

Following the KS estimator, Alkhamisi et al. suggested a list of estimators for k.[14]

$$AKS_1 = \max(s_i)\,;\, AKS_2 = \text{median}(s_i)\,;\, AKS_3 = mean(s_i);\, AKS_4 = \max\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i}\right) \tag{22}$$

where $s_i = \frac{\lambda_i\hat{\sigma}^2}{(n - p)\,\hat{\sigma}^2 + \lambda_i\hat{\alpha}_i^2}$ $(i=1, 2,..., p)$ and AKS_2 estimator is proposed for cases where the number of explanatory variables is greater than 3.

Alkhamisi and Shukur suggested the following estimators, taking advantage of a modification of the KS estimator.[15]

$$AS_1 = HKB + \frac{1}{\lambda_{max}}\,; \qquad AS_2 = HK2 + \frac{1}{\lambda_{max}}\,; \qquad AS_3 = LW + \frac{1}{\lambda_{max}}\,;$$

$$AS_4 = \max\left\{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2} + \frac{1}{\lambda_i}\right\}; \qquad AS_5 = \text{mean}\left\{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2} + \frac{1}{\lambda_i}\right\};$$

$$AS_6 = \text{median}\left\{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2} + \frac{1}{\lambda_i}\right\};$$

Then, Muniz and Kibria proposed the following estimators based on $m_{MunizKibria} = m_i = \sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}$.[16]

$$MK_1 = \max(m_i); \quad MK_2 = \max(1/m_i);$$

$$MK_3 = \text{median}(m_i); \quad MK_4 = \text{median}(1/m_i);$$

$$MK_5 = \text{geomean}(m_i) = \left(\prod_{i=1}^{p} m_i\right)^{1/p};$$

$$MK_6 = \text{geomean}(1/m_i) = \left(\prod_{i=1}^{p} \frac{1}{m_i}\right)^{1/p}$$

They also suggested the following estimator as another k estimator.

$$MK_7 = \left(\prod_{i=1}^{p} \frac{\lambda_i \hat{\sigma}^2}{(n-p)\,\hat{\sigma}^2 + \lambda_i \hat{\alpha}_i^2} \right)^{1/p} \tag{23}$$

Al-Hassan proposed the following estimator based on the AS estimators.[17]

$$Al\text{-}Hassan = \hat{\sigma}^2 \frac{\sum_{i=1}^{p} (\lambda_i \alpha_i)^2}{\left(\sum_{i=1}^{p} \lambda_i \alpha_i^2 \right)^2} + \frac{1}{\lambda_{max}} = HSL + \frac{1}{\lambda_{max}} \tag{24}$$

Muniz et al. suggested 5 new k estimators by using $v_i = \frac{\lambda_{max} \hat{\sigma}^2}{(n-p)\,\hat{\sigma}^2 + \lambda_{max} \hat{\alpha}_i^2}$.[18]

$\mathrm{MKMS}_1 = \max\,(1/\sqrt{v_i});$ $\mathrm{MKMS}_2 = \max\,(\sqrt{v_i});$ $\mathrm{MKMS}_3 = median\,(1/\sqrt{v_i});$

$$\mathrm{MKMS}_4 = \mathrm{geomean}\,(\sqrt{v_i}) = \left(\prod_{i=1}^{p} (\sqrt{v_i}) \right)^{1/p};$$

$$\mathrm{MKMS}_5 = \mathrm{geomean}\,(\sqrt{1/v_i}) = \left(\prod_{i=1}^{p} \frac{1}{(\sqrt{v_i})} \right)^{1/p}$$

Dorugade proposed new methods to determine ridge parameters k by adopting algorithms outlined Kibria.[19] The recommended estimators of ridge parameter k are as follows.

$$D_1 = \mathrm{mean}\left(\frac{2\hat{\sigma}^2}{\lambda_{max} \hat{\alpha}_i^2} \right); \qquad D_2 = \mathrm{median}\left(\frac{2\hat{\sigma}^2}{\lambda_{max} \hat{\alpha}_i^2} \right);$$

$$D_3 = \frac{2\hat{\sigma}^2}{\lambda_{max} \left(\prod_{i=1}^{p} \hat{\alpha}_i^2 \right)^{1/p}} \text{ and } D_4 = \frac{2p\hat{\sigma}^2}{\lambda_{max} \sum_{i=1}^{p} \hat{\alpha}_i^2}.$$

Karaibrahimoglu et al. suggested some estimators which were the modifications of D_1.[20]

$$KAG_1 = \frac{p}{\sqrt{\lambda_{max}}} \frac{\hat{\sigma}^2}{\sum_{i=1}^{p} \hat{\alpha}_i^2}; \qquad KAG_2 = \frac{\sqrt{5}p}{\lambda_{max}} \frac{\hat{\sigma}^2}{\sum_{i=1}^{p} \hat{\alpha}_i^2};$$

$$KAG_3 = \frac{2p}{\sum_{i=1}^{p} \left(\lambda_i^{1/4} \right)} \frac{\hat{\sigma}^2}{\sum_{i=1}^{p} \hat{\alpha}_i^2}; \qquad KAG_4 = \frac{2p}{\sqrt{\sum_{i=1}^{p} \lambda_i}} \frac{\hat{\sigma}^2}{\sum_{i=1}^{p} \hat{\alpha}_i^2}.$$

Finally, Asar and Genc defined a new quantity as a modification of LW.[21] Based on $d_i = \sqrt{\frac{\hat{\sigma}^2}{\lambda_i \hat{\alpha}_i^2}}$, they proposed 9 new ridge parameter estimators.

$AG_1 = \max(d_i)$; $AG_2 = \max(1/d_i)$; $AG_3 = median(d_i)$; $AG_4 = median(1/d_i)$;

$AG_5 = \text{mean}(d_i)$; $AG_6 = \text{mean}(1/d_i)$; $AG_7 = geomean(d_i) = \left(\prod \frac{1}{d_i}\right)^{1/p}$

$AG_8 = harmmean(d_i) = \frac{p}{\sum_{i=1}^{p} d_i}$ and $AG_9 = harmmean\left(\frac{1}{d_i}\right) = \frac{p}{\sum_{i=1}^{p} \frac{1}{d_i}}$.

By adopting algorithms outlined in Kibria and Asar and Genc, we suggest a new estimator to determine ridge parameter in Equation (25).[12,21] The suggested estimator is obtained by multiplying $\left(\sum_{i=1}^{n} \lambda_i\right)\hat{\alpha}_i^2$ with the square root of the reciprocal m_{Kibria} which is $\sqrt{\frac{\hat{\alpha}_i^2}{\hat{\sigma}^2}}$. We have $\left(\sum_{i=1}^{n} \lambda_i\right) > p$ because the matrix $X'X$ is in the correlation form.

$$\text{AAY} = \max \sqrt{\frac{\left(\sum_{i=1}^{n} \lambda_i\right)\hat{\sigma}_i^2\,\hat{\alpha}_i^2}{\hat{\sigma}^2}} \tag{25}$$

RESULTS

The aim of this article is to compare the performance of our new proposed estimator with both the OLS and the other estimators. In this study, a simulation has been conducted to compare the performances of the given estimators using R language since a theoretical comparison is not possible. A well-designed Monte Carlo simulation depends on the factors under which the estimators are addressed, and it also depends on which criteria to be used to compare the estimators. In this study, there are 4 factors affecting the estimators. These are the degrees of multicollinearity, the sample size, the number of explanatory variables, and the population variance. In order to get different degrees of multicollinearity and to generate the explanatory variables, we used the following generally used expression:

$$x_{ij} = (1-\gamma^2)^{1/2} z_{ij} + \gamma z_{ip} \tag{26}$$

where $i = 1,2,\ldots,n$ and $j = 1,2,\ldots,p$ and γ^2 represents the correlation between the explanatory variables, $z_{ij}'s$ are independent standart normal pseudo-random numbers and ε_i has a normal distribution with zero mean and variance σ^2.[21] The n observations for dependent variable are generated by

$$y_i = \beta_1 x_{i1} + \ldots + \beta_p x_{ip} + \varepsilon_i \; i = 1,2,\ldots,n. \tag{27}$$

Our primary interest is to measure the performance of our proposed estimator according to the strength of the multicollinearity, so, we consi9dered four different degrees of correlation (γ=0.70, 0.80, 0.90, 0,95). Since we want to measure the effect of the sample size on the performance of the estimators, four different values of the sample size n=20, 50, 70, 100 were taken into consideration. These sample sizes include models with small, medium and large sizes. Since the bad impact of the collinearity on the MSE might be stronger when the number of correlated variables in the model is large, the number of explanatory variables is also of great importance. Therefore, the number of explanatory variables in the model is taken as 4 and 6. For each set of explanatory variables, we considered the coefficient vector that corresponded to the largest eigenvalue of $X'X$ matrix subject to the constraint that $\beta'\beta=1$. Kibria has reported that the largest eigenvector of $X'X$ can be used as a regression coefficient in order to get a minimized MSE value.[12]

Newhouse and Oman stated that if the MSE is a function of β , σ^2 and k, and if the explanatory variables are fixed, then the MSE is minimized when we choose this coefficient vector.[22] Finally, the error variance is chosen as $\sigma^2=1$. For different values of the sample size, correlation degrees and explanatory variable number, the simulation was repeated 2000 times and the MSE was calculated for all 55 estimators. The relative efficiencies of the new proposed estimator relative to all estimators was calculated for convenience when comparing MSE values. For each estimator we compute the relative efficiency (RE) values by using the following equation

$$RE_{proposed} = \frac{MSE\,(k_{proposed})}{MSE\,(k_{other})}.$$

The simulation results are given in Table 1-4.

TABLE 1: Relative efficiency values for the estimators (p=4).

γ	0.7				0.8			
Estimators	**20**	**50**	**70**	**100**	**20**	**50**	**70**	**100**
OLS	0.1546	0.1690	0.3771	0.4363	0.1960	0.2348	0.3940	0.4556
HK1	0.1817	0.2109	0.4694	0.5205	0.2063	0.3390	0.4095	0.4530
HK2	0.1813	0.2107	0.4696	0.5207	0.2087	0.3388	0.4091	0.4530
HK3	0.1875	0.2530	0.4651	0.5283	0.2110	0.3480	0.4126	0.4694
THEO	0.2295	0.4368	0.5930	0.5401	0.2849	0.4017	0.4515	0.5034
HKB	0.1951	0.2633	0.4777	0.5330	0.2209	0.3700	0.4100	0.5020
LW	0.1827	0.2704	0.4878	0.5367	0.2023	0.3360	0.3947	0.4560
HSL	0.2226	0.4579	0.5677	0.6196	0.2702	0.3760	0.4277	0.4821
Kibria1	0.1885	0.3998	0.4422	0.5208	0.2985	0.3986	0.4246	0.4612
Kibria2	0.1916	0.3602	0.4510	0.5378	0.3201	0.3882	0.4661	0.5469
Kibria3	0.2179	0.4263	0.4612	0.5342	0.5002	0.5313	0.5430	0.5555
KS	0.2185	0.4992	0.5603	0.6136	0.2579	0.3677	0.4203	0.4760
AKS1	0.2530	0.5015	0.5613	0.6140	0.2672	0.3679	0.4204	0.4760
AKS2	0.2099	0.4930	0.5553	0.6099	0.2356	0.3531	0.4081	0.4664
AKS3	0.2122	0.4930	0.5554	0.6100	0.2379	0.3549	0.4099	0.4680
AKS4	0.4537	0.5098	0.5128	0.5436	0.4138	0.4651	0.5098	0.5371
AS1	0.3565	0.6037	0.6479	0.6831	0.3729	0.4704	0.5102	0.5521
AS2	0.2308	0.5112	0.5696	0.6206	0.2782	0.3793	0.4296	0.4831
AS3	0.1918	0.4707	0.5380	0.5968	0.2041	0.3363	0.3948	0.4561
AS4	0.2094	0.2189	0.4927	0.5398	0.2314	0.3406	0.4211	0.4870
AS5	0.2987	0.3681	0.4522	0.5208	0.3000	0.3986	0.4747	0.4912
AS6	0.3106	0.3603	0.4710	0.5418	0.3210	0.3888	0.4063	0.4769
MK1	0.4732	0.4983	0.5165	0.5405	0.4930	0.5043	0.5166	0.5241
MK2	0.2499	0.5138	0.5711	0.6219	0.3017	0.3805	0.4295	0.4831
MK3	0.3905	0.6513	0.6847	0.7324	0.4016	0.5057	0.5484	0.6080
MK4	0.1931	0.4819	0.5453	0.6012	0.2501	0.3508	0.4046	0.4622
MK5	0.4109	0.6463	0.6816	0.7252	0.4142	0.5011	0.5405	0.5894

TABLE 1: Relative efficiency values for the estimators (p=4).

γ	0.7				0.8			
Estimators	**20**	**50**	**70**	**100**	**20**	**50**	**70**	**100**
MK6	0.1882	0.4802	0.5448	0.6013	0.2374	0.3484	0.4035	0.4619
MK7	0.2045	0.4920	0.5547	0.6096	0.2318	0.3532	0.4087	0.4671
Al-Hassan	0.2238	0.5083	0.5679	0.6197	0.2720	0.3763	0.4278	0.4822
MKMS1	0.4570	0.5613	0.5967	0.7560	0.4535	0.5074	0.5750	0.5873
MKMS2	0.3243	0.5385	0.5896	0.6354	0.3566	0.4125	0.4550	0.5028
MKMS3	0.2109	0.4993	0.5596	0.6128	0.2735	0.3656	0.4172	0.4728
MKMS4	0.2735	0.5247	0.5801	0.6288	0.3070	0.3945	0.4420	0.4936
MKMS5	0.2155	0.5026	0.5624	0.6151	0.2734	0.3683	0.4198	0.4752
D1	0.2667	0.4052	0.4137	0.5356	0.2558	0.3296	0.3995	0.3973
D2	0.1951	0.4911	0.5459	0.5971	0.2265	0.3506	0.4039	0.4678
D3	0.1949	0.4864	0.5477	0.6047	0.2213	0.3469	0.4025	0.4617
D4	0.1908	0.4713	0.5384	0.5971	0.2030	0.3367	0.3951	0.4563
KAG1	0.1889	0.4814	0.5457	0.6020	0.2207	0.3459	0.4020	0.4612
KAG2	0.1915	0.4716	0.5386	0.5972	0.2039	0.3369	0.3953	0.4564
KAG3	0.1995	0.4964	0.5578	0.6111	0.2446	0.3627	0.4159	0.4723
KAG4	0.2012	0.4885	0.5507	0.6052	0.2377	0.3538	0.4077	0.4651
AG1	0.4405	0.5068	0.5384	0.6182	0.4057	0.4311	0.4450	0.5212
AG2	0.8265	0.9308	0.9386	0.9490	0.9101	0.9192	0.9281	0.9356
AG3	0.2454	0.5121	0.5697	0.6218	0.2851	0.3806	0.4286	0.4842
AG4	0.4528	0.5390	0.5827	0.6395	0.4032	0.4275	0.4446	0.5131
AG5	0.4680	0.5535	0.5987	0.6421	0.4234	0.4459	0.4818	0.5472
AG6	0.4846	0.6292	0.6746	0.7642	0.4898	0.5157	0.5567	0.5990
AG7	0.2394	0.5014	0.5601	0.6134	0.2673	0.3670	0.4178	0.4737
AG8	0.2073	0.5052	0.5655	0.6176	0.2712	0.3743	0.4268	0.4802
AG9	0.1979	0.4809	0.5451	0.6016	0.2297	0.3463	0.4019	0.4610

TABLE 2: Relative efficiency values for the estimators (p=4) (Continue).

γ	0.9				0.95			
n	**20**	**50**	**70**	**100**	**20**	**50**	**70**	**100**
OLS	0.0769	0.1435	0.1951	0.2476	0.0333	0.0697	0.0891	0.1155
HK1	0.0804	0.1781	0.2013	0.2574	0.0350	0.0706	0.0908	0.1170
HK2	0.0816	0.1798	0.2052	0.2573	0.0333	0.0749	0.0915	0.1178
HK3	0.0826	0.1823	0.2105	0.2612	0.0380	0.0771	0.0928	0.1196
THEO	0.1283	0.2321	0.2567	0.3054	0.0626	0.1052	0.1259	0.1507
HKB	0.0846	0.1909	0.2118	0.2659	0.0433	0.0840	0.0968	0.1198
LW	0.0812	0.1745	0.2057	0.2579	0.0366	0.0705	0.0895	0.1158
HSL	0.1395	0.2158	0.2398	0.2866	0.0867	0.1047	0.1193	0.1415
Kibria1	0.1660	0.2292	0.2317	0.2648	0.1646	0.1906	0.2060	0.2158
Kibria2	0.2460	0.3864	0.4332	0.4368	0.1321	0.1995	0.2503	0.2877
Kibria3	0.3017	0.4677	0.4937	0.5586	0.1568	0.2409	0.2986	0.3306
KS	0.1246	0.2080	0.2330	0.2806	0.0678	0.0965	0.1132	0.1363

TABLE 2: Relative efficiency values for the estimators (*p*=4) (Continue).

γ	0.9				0.95			
n	20	50	70	100	20	50	70	100
AKS1	0.1348	0.2081	0.2331	0.2806	0.0827	0.0987	0.1137	0.1364
AKS2	0.0948	0.1830	0.2126	0.2637	0.0408	0.0736	0.0924	0.1183
AKS3	0.1024	0.1883	0.2170	0.2674	0.0499	0.0793	0.0973	0.1226
AKS4	0.2433	0.3363	0.3596	0.4188	0.1515	0.1808	0.2018	0.2356
AS1	0.1832	0.2952	0.3120	0.3560	0.0946	0.1443	0.1669	0.1892
AS2	0.1367	0.2184	0.2415	0.2875	0.0752	0.1039	0.1200	0.1421
AS3	0.0824	0.1748	0.2058	0.2580	0.0374	0.0707	0.0896	0.1159
AS4	0.1107	0.1625	0.2376	0.2607	0.1456	0.1515	0.1579	0.1684
AS5	0.1623	0.2296	0.2319	0.2660	0.1937	0.1949	0.2064	0.2075
AS6	0.2728	0.3909	0.4256	0.4375	0.1843	0.2092	0.2570	0.2912
MK1	0.3924	0.4585	0.4592	0.4929	0.2363	0.2856	0.3262	0.3351
MK2	0.1758	0.2194	0.2421	0.2874	0.1030	0.1116	0.1218	0.1428
MK3	0.2053	0.3066	0.3293	0.3856	0.1193	0.1482	0.1716	0.1971
MK4	0.1307	0.1941	0.2196	0.2670	0.0875	0.0925	0.1057	0.1278
MK5	0.2235	0.3128	0.3277	0.3799	0.1318	0.1578	0.1789	0.1996
MK6	0.1183	0.1900	0.2173	0.2658	0.0760	0.0871	0.1018	0.1254
MK7	0.0950	0.1850	0.2144	0.2653	0.0424	0.0753	0.0939	0.1197
Al-Hassan	0.1408	0.2161	0.2399	0.2867	0.0878	0.1050	0.1194	0.1415
MKMS1	0.4839	0.5256	0.5468	0.5913	0.4198	0.4361	0.4562	0.4664
MKMS2	0.2202	0.2583	0.2728	0.3129	0.1405	0.1496	0.1510	0.1671
MKMS3	0.1452	0.2058	0.2298	0.2762	0.0989	0.1004	0.1131	0.1346
MKMS4	0.1643	0.2343	0.2552	0.3000	0.1002	0.1162	0.1314	0.1524
MKMS5	0.1437	0.2071	0.2315	0.2783	0.0974	0.1000	0.1133	0.1353
D1	0.1107	0.2018	0.2188	0.2743	0.0488	0.0847	0.1038	0.1274
D2	0.0873	0.1815	0.2111	0.2645	0.0378	0.0737	0.0922	0.1185
D3	0.0869	0.1797	0.2094	0.2613	0.0382	0.0727	0.0913	0.1174
D4	0.0862	0.1749	0.2059	0.2581	0.0351	0.0705	0.0896	0.1159
KAG1	0.0897	0.1823	0.2116	0.2626	0.0403	0.0748	0.0935	0.1191
KAG2	0.0895	0.1750	0.2060	0.2582	0.0353	0.0706	0.0897	0.1160
KAG3	0.1068	0.1997	0.2262	0.2755	0.0518	0.0870	0.1053	0.1298
KAG4	0.1004	0.1898	0.2172	0.2670	0.0467	0.0796	0.0976	0.1224
AG1	0.2787	0.3157	0.3187	0.3729	0.1808	0.1832	0.1987	0.2058
AG2	0.8763	0.9472	0.9460	0.9468	0.8782	0.9258	0.9616	0.9647
AG3	0.1448	0.2180	0.2399	0.2881	0.0905	0.1053	0.1204	0.1419
AG4	0.1809	0.2344	0.2561	0.2977	0.1196	0.1220	0.1325	0.1540
AG5	0.4792	0.5069	0.5115	0.5354	0.4153	04251	0.4413	0.4586
AG6	0.4933	0.5394	0.5544	0.6027	0.4623	0.4743	0.4885	0.5085
AG7	0.1319	0.2049	0.2286	0.2772	0.0787	0.0946	0.1100	0.1322
AG8	0.1339	0.2108	0.2376	0.2824	0.0776	0.0997	0.1141	0.1393
AG9	0.1026	0.1843	0.2127	0.2630	0.0539	0.0782	0.0954	0.1201

TABLE 3: Relative efficiency values for the estimators (p=6) .

γ	0.7				0.8			
n	20	50	70	100	20	50	70	100
OLS	0.1734	0.3495	0.4138	0.4785	0.1091	0.2470	0.2913	0.3472
HK1	0.1781	0.3550	0.4145	0.4793	0.1181	0.2431	0.3308	0.3501
HK2	0.1795	0.3550	0.4149	0.4805	0.1192	0.2430	0.3005	0.3500
HK3	0.1806	0.3563	0.4154	0.4868	0.1255	0.2448	0.3035	0.3562
THEO	0.1942	0.3832	0.4396	0.4951	0.1509	0.2672	0.3061	0.3654
HKB	0.1829	0.3529	0.4152	0.4896	0.1225	0.2428	0.3400	0.3582
LW	0.1802	0.3507	0.4145	0.4799	0.1143	0.2480	0.3019	0.3576
HSL	0.2364	0.3825	0.4401	0.4988	0.1666	0.2809	0.3294	0.3794
Kibria1	0.3007	0.3885	0.4468	0.4886	0.2368	0.2751	0.3235	0.3717
Kibria2	0.3396	0.4544	0.4984	0.5225	0.4410	0.6185	0.6390	0.6496
Kibria3	0.5369	0.6067	0.6376	0.6465	0.5264	0.7244	0.7483	0.7558
KS	0.2309	0.3775	0.4354	0.4949	0.1595	0.2764	0.3252	0.3758
AKS1	0.2635	0.3809	0.4370	0.4955	0.1764	0.2768	0.3253	0.3759
AKS2	0.2199	0.3700	0.4294	0.4902	0.1415	0.2618	0.3130	0.3661
AKS3	0.2226	0.3703	0.4297	0.4904	0.1442	0.2634	0.3143	0.3673
AKS4	0.4750	0.5363	0.5590	0.5603	0.4048	0.4990	0.5271	0.5491
AS1	0.3795	0.5131	0.5533	0.5897	0.2636	0.4030	0.4401	0.4752
AS2	0.2443	0.3853	0.4416	0.4996	0.1691	0.2833	0.3309	0.3802
AS3	0.1813	0.3510	0.4146	0.4898	0.1152	0.2488	0.3020	0.3576
AS4	0.1834	0.3979	0.4189	0.4806	0.2080	0.2597	0.3132	0.3681
AS5	0.1809	0.3585	0.4168	0.4886	0.1677	0.2551	0.3135	0.3617
AS6	0.4271	0.4946	0.5585	0.5625	0.4581	0.6199	0.6395	0.6498
MK1	0.4675	0.4916	0.5469	0.5724	0.5409	0.5703	0.5979	0.6086
MK2	0.2605	0.3863	0.4424	0.5000	0.1947	0.2841	0.3309	0.3801
MK3	0.3998	0.5472	0.6027	0.6440	0.2900	0.4223	0.4721	0.5126
MK4	0.2031	0.3574	0.4189	0.4818	0.1411	0.2564	0.3077	0.3613
MK5	0.4213	0.5471	0.5989	0.6377	0.3142	0.4288	0.4703	0.5103
MK6	0.1981	0.3568	0.4285	0.4817	0.1353	0.2553	0.3072	0.3611
MK7	0.2145	0.3690	0.4289	0.4900	0.1376	0.2618	0.3132	0.3665
Al-Hassan	0.2376	0.3827	0.4402	0.4988	0.1675	0.2811	0.3295	0.3795
MKMS1	0.5258	0.5307	0.5458	0.5627	0.6006	0.6179	0.6341	0.6425
MKMS2	0.3365	0.4206	0.4680	0.5189	0.2747	0.3266	0.3641	0.4055
MKMS3	0.2204	0.3696	0.4292	0.4901	0.1546	0.2665	0.3167	0.3688
MKMS4	0.2864	0.4065	0.4587	0.5126	0.2127	0.3077	0.3509	0.3965
MKMS5	0.2251	0.3722	0.4312	0.4916	0.1578	0.2690	0.3187	0.3704
D1	0.2735	0.3754	0.4263	0.4805	0.1901	0.2953	0.3372	0.3597
D2	0.2043	0.3689	0.4299	0.4910	0.1266	0.2580	0.3117	0.3642
D3	0.2056	0.3655	0.4265	0.4878	0.1269	0.2572	0.3094	0.3631
D4	0.1802	0.3514	0.4250	0.4896	0.1133	0.2487	0.3027	0.3587
KAG1	0.2000	0.3618	0.4226	0.4844	0.1270	0.2573	0.3091	0.3628

TABLE 3: Relative efficiency values for the estimators (p=6) .

γ	0.7				0.8			
n	20	50	70	100	20	50	70	100
KAG2	0.1810	0.3517	0.4151	0.4892	0.1138	0.2489	0.3024	0.3588
KAG3	0.2117	0.3714	0.4309	0.4910	0.1385	0.2684	0.3189	0.3709
KAG4	0.2136	0.3682	0.4272	0.4875	0.1386	0.2643	0.3144	0.3665
AG1	0.4529	0.5132	0.5362	0.5636	0.4039	0.4374	0.4669	0.4711
AG2	0.8263	0.8627	0.8784	0.8890	0.8283	0.8810	0.8895	0.8954
AG3	0.2571	0.3917	0.4475	0.5034	0.1839	0.2897	0.3365	0.3838
AG4	0.2522	0.3876	0.4427	0.5014	0.1835	0.2849	0.3321	0.3812
AG5	0.5198	0.5315	0.5333	0.5546	0.5255	0.5346	0.5444	0.5605
AG6	0.5436	0.5511	0.5611	0.5932	0.5800	0.5902	0.6034	0.6267
AG7	0.2511	0.3850	0.4415	0.4986	0.2781	0.3826	0.3293	0.3782
AG8	0.1877	0.3514	0.4301	0.4919	0.1191	0.2684	0.2791	0.3510
AG9	0.2086	0.3614	0.4220	0.4839	0.1393	0.2582	0.3098	0.3626

TABLE 4: Relative efficiency values for the estimators (p=6) (Continue).

γ	0.9				0.95			
n	20	50	70	100	20	50	70	100
OLS	0.0432	0.1167	0.1504	0.1879	0.0155	0.0407	0.0599	0.0805
HK1	0.0787	0.1501	0.1788	0.2108	0.0155	0.0466	0.0720	0.0824
HK2	0.0848	0.1503	0.1787	0.2107	0.0169	0.0474	0.0722	0.0904
HK3	0.0564	0.1350	0.1684	0.2046	0.0173	0.0483	0.0735	0.0919
THEO	0.0693	0.1539	0.1869	0.2218	0.0199	0.0632	0.0816	0.1039
HKB	0.1228	0.2362	0.2665	0.2951	0.0183	0.0571	0.0787	0.0958
LW	0.0464	0.1175	0.1519	0.1882	0.0177	0.0482	0.0602	0.0877
HSL	0.0864	0.1490	0.1777	0.2102	0.0479	0.0677	0.0818	0.1000
Kibria1	0.0589	0.1662	0.1830	0.1959	0.1670	0.2102	0.2315	0.2362
Kibria2	0.2127	0.4742	0.5383	0.6024	0.0896	0.1856	0.2410	0.3326
Kibria3	0.2658	0.5560	0.6207	0.7015	0.3204	0.5660	0.6497	0.7126
KS	0.0748	0.1447	0.1741	0.2070	0.0321	0.0633	0.0786	0.0973
AKS1	0.0868	0.1454	0.1741	0.2070	0.0481	0.0652	0.0791	0.0974
AKS2	0.0561	0.1239	0.1563	0.1927	0.0204	0.0463	0.0622	0.0826
AKS3	0.0604	0.1270	0.1589	0.1948	0.0248	0.0492	0.0648	0.0849
AKS4	0.2266	0.3763	0.3918	0.4207	0.1252	0.1798	0.1991	0.2504
AS1	0.1237	0.2364	0.2666	0.2952	0.0509	0.1072	0.1308	0.1568
AS2	0.0794	0.1502	0.1789	0.2108	0.0340	0.0667	0.0820	0.1004
AS3	0.0470	0.1176	0.1510	0.1883	0.0181	0.0443	0.0603	0.0808
AS4	0.1167	0.1504	0.1601	0.1948	0.1451	0.1515	0.1595	0.1610
AS5	0.1669	0.2565	0.2631	0.2959	0.1965	0.2128	0.2328	0.2366
AS6	0.2476	0.4808	0.5419	0.6039	0.1397	0.1973	0.2483	0.3369
MK1	0.4078	0.5422	0.5511	0.5534	0.4273	0.5566	0.5769	0.5897
MK2	0.1170	0.1524	0.1794	0.2111	0.0671	0.0730	0.0838	0.1007

TABLE 4: Relative efficiency values for the estimators (p=6) (Continue).								
γ	0.9				0.95			
n	20	50	70	100	20	50	70	100
MK3	0.1452	0.2387	0.2681	0.3111	0.0749	0.1080	0.1271	0.1551
MK4	0.0738	0.1284	0.1590	0.1934	0.0441	0.0559	0.0693	0.0871
MK5	0.1627	0.2485	0.2764	0.3147	0.0878	0.1184	0.1364	0.1602
MK6	0.0680	0.1266	0.1578	0.1928	0.0379	0.0535	0.0676	0.0863
MK7	0.0556	0.1247	0.1572	0.1935	0.0208	0.0470	0.0629	0.0832
Al-Hassan	0.0870	0.1491	0.1778	0.2102	0.0484	0.0678	0.0818	0.1000
MKMS1	0.4202	0.5550	0.5719	0.6134	0.4309	0.5851	0.5958	0.6325
MKMS2	0.3796	0.3990	0.4177	0.4421	0.4041	0.4191	0.4272	0.4598
MKMS3	0.2813	0.3055	0.3256	0.3494	0.2995	0.3102	0.3335	0.3613
MKMS4	0.3157	0.3730	0.3991	0.4290	0.3340	0.3829	0.3968	0.4346
MKMS5	0.3823	0.3974	0.4173	0.4310	0.4094	0.4108	0.4143	0.4424
D1	0.0735	0.1520	0.1836	0.2151	0.0278	0.0573	0.0732	0.0934
D2	0.0493	0.1214	0.1544	0.1917	0.0179	0.0454	0.0613	0.0819
D3	0.0499	0.1210	0.1540	0.1910	0.0182	0.0454	0.0614	0.0818
D4	0.0450	0.1176	0.1511	0.1883	0.0163	0.0441	0.0602	0.0808
KAG1	0.0514	0.1236	0.1562	0.1924	0.0190	0.0471	0.0631	0.0834
KAG2	0.0452	0.1177	0.1511	0.1884	0.0163	0.0441	0.0603	0.0808
KAG3	0.0594	0.1341	0.1661	0.2012	0.0233	0.0538	0.0702	0.0906
KAG4	0.0581	0.1295	0.1610	0.1961	0.0220	0.0504	0.0661	0.0861
AG1	0.2666	0.3104	0.3157	0.3364	0.2952	0.3269	0.3382	0.3535
AG2	0.8059	0.9128	0.9245	0.9257	0.8012	0.8972	0.9395	0.9547
AG3	0.0990	0.1561	0.1828	0.2161	0.1075	0.1725	0.1850	0.2138
AG4	0.0969	0.1512	0.1807	0.2113	0.1057	0.1702	0.1842	0.2108
AG5	0.4391	0.4643	0.5171	0.5786	0.4903	0.5373	0.5575	0.5745
AG6	0.4865	0.4942	0.5525	0.6312	0.4900	0.5935	0.6307	0.6896
AG7	0.0927	0.1486	0.1763	0.2095	0.0932	0.1571	0.1799	0.2180
AG8	0.0722	0.1359	0.1677	0.2009	0.0840	0.1581	0.1737	0.2127
AG9	0.0641	0.1264	0.1575	0.1929	0.0704	0.1305	0.1653	0.1946

DISCUSSION

The results in Monte Carlo studies based on the estimation of the ridge parameter are usually tabulated for MSE values of the estimators under different conditions. However, the number of estimators in this study is 55. When comparing these estimators with the new proposed estimator, it is difficult to follow the MSE values. Therefore, Table 1-4 gives the RE values which are calculated as the ratio of the MSE value of the new proposed estimator to the MSE value of the other estimators. The RE values are given in Table 1-2 for different sample size, correlation degrees and 4 explanatory variables. Similarly, Table 3-4 shows the results for 6 explanatory variables. Table 1 shows that the most efficient estimator is the proposed estimator when the correlation between the explanatory variables is 0.70 and the sample size is 20. Then, AG2, AG6, MK1, AG5, AKS4 and MK5 follow it. The estimator that gives the worst result under these simulation conditions is OLS. Estimators with the least efficiency after OLS are HK2, LW, HK1 and HK3.

If the sample size is changed to 50, since all the RE values are still less than 1, the proposed estimator is again the most efficient one. AG2, AG6, MK3, MK5, AS1 and MKMS1 estimators come after that. When the sample volume is changed to 70, the best estimators are listed as AAY, AG2, MK3, MK5 and AG6. In this case, OLS estimator efficiency is the worst, followed by HK1, HK2, HK3, HKB and LW, respectively. In the case of $n = 100$, the most effective estimators are AAY, AG2, AG6, MKMS1, MK5, MK3, AS1 and AG5.

When the number of explanatory variables is 4, the most effective estimator in each sample size is again the proposed AAY estimator when the correlation degree is taken as 0.8. Then, AG2 estimator comes after it. Other efficient estimators are Kibria3, MK1, AG6, AG5, MKMS1 and MK5, although the sequence changes due to increased sample size. The least effective estimators were obtained as OLS, LW, D4, KAG2, AS3, HK1, HK2, HK3, HKB.

When the degree of correlation is taken as 0.90, AAY and AG2 are the most effective estimators, respectively. These are followed by AG6, MKMS1, AG5, Kibria3 and MK1 estimators. If the correlation degree is increased to 0.95, the order of AAY and AG2 estimators does not change. However, the order of AG6, MKMS1, AG5, MK1 and Kibria3 estimators varies depending on the sample size.

Table 3-4 shows that AAY and AG2 are the most effective estimators for all correlation degrees and sample sizes, respectively. These are followed by Kibria3, MK5, AG6, AKS3, AKS4 and MKMS1 estimators. The OLS estimator efficiency in all cases is the worst.

Table 1-4 shows that all the values of RE are smaller than 1. These results show that the new proposed estimator yields more effective results than the other estimators discussed.

REAL DATA APPLICATION

In this section, we have given a real-life data application to demonstrate the performance of the proposed estimator. We use the actual data set originally worked by Stamey et al.[23] The dependent and independent variables in the data set are as follows.

X1 Log Cancer volume (lcavol)

X2 Log Prostate weight (lweight)

X3 Log Bening prostatic hyperplasia amount (lpbh)

X4 Age

X5 Log capsular penetration (lcp)

X6 Seminal vesicle invasion (svi)

X7 Gleason score (gleason)

X8 Percent of Gleason score 4 or 5

Y Log Prostate specific antigen (lpsa)

The largest and smallest eigenvalues of the matrix $X X'$ are obtained as 479080 and 8.0931, respectively. The condition number of the data is 59196. The magnitude of the condition number indicates the presence of the multicollinearity. MSE values of the estimators discussed in the study were calculated and given in Table 5. According to Table 5, AAY and AG2 are the estimators with the smallest MSE.

TABLE 5: MSE values of the estimators.

OLS	**HK1**	**HK2**	**HK3**	**THEO**	**HKB**	**LW**	**HSL**	**Kibria1**
0,11015	0,12056	0,10142	0,10421	0,10064	0,11731	0,11005	0,01828	0,06150
Kibria2	**Kibria3**	**KS**	**AKS1**	**AKS2**	**AKS3**	**AKS4**	**AS1**	**AS2**
0,09565	0,05785	0,09950	0,00543	0,09930	0,02608	0,00614	0,09588	0,09950
AS3	**AS4**	**AS5**	**AS6**	**MK1**	**MK2**	**MK3**	**MK4**	**MK5**
0,11005	0,01476	0,01498	0,09564	0,00615	0,10048	0,07493	0,10763	0,08006
MK6	**MK7**	**Al-Hassan**	**MKMS1**	**MKMS2**	**MKMS3**	**MKMS4**	**MKMS5**	**D1**
0,10895	0,08996	0,01828	0,10088	0,04352	0,10333	0,09540	0,10640	0,10140
D2	**D3**	**D4**	**KAG1**	**KAG2**	**KAG3**	**KAG4**	**AG1**	**AG2**
0,11015	0,11015	0,11015	0,11007	0,11015	0,10823	0,10999	0,13782	0,48129
AG3	**AG4**	**AG5**	**AG6**	**AG7**	**AG8**	**AG9**	**AAY**	
0,10552	0,11760	0,10133	0,18193	0,10548	0,10786	0,10073	0,00264	

CONCLUSION

In this study, we proposed a new ridge parameter estimator which is obtained as a modified version of the estimator Kibria and Asir and Genc in the presence of multicollinearity.[12,26] In order to compare the proposed estimator with OLS and other estimators in the literature, we performed a Monte Carlo simulation based on the different correlation levels, sample sizes and number of explanatory variables. In the experiments, we made 2000 replications for all combinations of these factors. For all estimators, first we determined the MSE values. Due to the large number of estimators included in the study, we have calculated the RE values to determine the performance of the proposed estimator.

When the RE values of all estimators were examined, it was found that all of them were less than 1. This shows that the new proposed estimator gives more effective results than the other estimators discussed. The most effective estimators after the proposed estimator are the estimators with the highest RE. These estimators were determined as AG6, MKMS1, AG5, Kibria3 and MK1. Estimators with the smallest RE value are the least effective estimators. The least effective estimators is OLS and followed by HK1, HK2, HK3, HKB, D4, KAG2, AS3 and LW. As a result, the proposed estimator showed a higher performance in all cases than the other estimators. We can conclude that the proposed estimator is useful and it may be recommended to the practitioners.

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Conflict of Interest

No conflicts of interest between the authors and / or family members of the scientific and medical committee members or members of the potential conflicts of interest, counseling, expertise, working conditions, share holding and similar situations in any firm.

Authorship Contributions

This study is entirely author's own work and no other author contribution.

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